

$B_{s,d}^* \rightarrow \mu^+ \mu^-$ and its impact on $B_{s,d} \rightarrow \mu^+ \mu^-$

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ABSTRACT: The decay of $B_{s,d}^* \rightarrow \mu^+ \mu^-$ and its impact on $B_{s,d} \rightarrow \mu^+ \mu^-$ is studied here. The $\mu^+ \mu^-$ decay widths of vector mesons $B_{s,d}^*$ are about a factor of 700 larger than the corresponding scalar mesons $B_{s,d}$. The obtained ratio of the branching fractions $Br(B_{s,d}^* \rightarrow \mu^+ \mu^-)/Br(B_{s,d} \rightarrow \mu^+ \mu^-)$ is about $\frac{0.3 \times \text{eV}}{\Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma)}$. At the same time, the hadronic contribution $B_{s,d} \rightarrow B_{s,d}^* \gamma \rightarrow \mu^+ \mu^-$ is estimated too. The relative increase of the amplitude of $B_{s,d} \rightarrow \mu^+ \mu^-$ is about $(0.01 \pm 0.006) \sqrt{\frac{\Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma)}{100 \text{ eV}}}$. If we choose $\Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma) = 2 \text{ eV}$, the branching fractions of the vector mesons to lepton pair are $(6.2 \pm 0.6) \times 10^{-10}$ and $(1.7 \pm 0.2) \times 10^{-11}$ for B_s^* and B_d^* respectively. If we choose $\Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma) = 200 \text{ eV}$, the updated branching fractions of the scalar mesons to muon pair are $(3.78 \pm 0.25) \times 10^{-9}$ and $(1.09 \pm 0.09) \times 10^{-10}$ for B_s and B_d respectively. Further studies on $B_{s,d}^*$ are usefully here, including dielectron decay, two-body decay with J/ψ , and so on.

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1 Introduction

The leptonic decays of the $B_{s,d}$ mesons play an important role in the standard model (SM) and the new physics (NP) [1, 2]. They are highly suppressed in the SM since the flavor changing neutral current decays are generated through W-box and Z-penguin diagrams. Furthermore, the branching fractions of leptonic decays of scalar meson undergo an additional helicity suppression factor by m_μ^2/M_S^2 , where m_μ and M_S denote masses of the muon lepton and the scalar meson, respectively. The suppression factor can be removed in some NP models, such as the two-Higgs doublet models [3], the minimal supersymmetric standard model (MSSM) [4], the next minimal supersymmetric standard model (NMSSM) [5], the dark matter [6], the universal extra dimensional model [7], lepton universality violation model [8], the four generations fermion [9], and so on [10]. The branching fractions of $B_{s,d} \rightarrow \mu^+\mu^-$ measured by the CMS and LHCb Collaborations [2] and predicted within the SM [1] with NNLO QCD [11] and NLO EW [12] corrections included are collected in Table 1.

Since the experimental branching fractions of $B_{s,d} \rightarrow \mu^+\mu^-$ are measured from the dimuon distributions by the CMS and LHCb Collaborations [2], the process $B_{s,d}^* \rightarrow \mu^+\mu^-$ will enhance the dimuon distributions for the mass splitting are about 45 MeV between $B_{s,d}$ and $B_{s,d}^*$. In another hand, the hadronic contribution $B_{s,d} \rightarrow B_{s,d}^*\gamma \rightarrow \mu^+\mu^-$ are missed in the theoretical prediction [1]. So we study $B_{s,d}^* \rightarrow \mu^+\mu^-$ and its impact on $B_{s,d} \rightarrow \mu^+\mu^-$ within SM here. The process $B_s \rightarrow B_s^*\gamma \rightarrow \mu^+\mu^-\gamma$ was considered in Ref. [13]. Recently, $B_{s,d}^* \rightarrow \mu^+\mu^-$ are considered in Ref. [14, 15]. And hadronic contribution from charmonium in $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow X_s\gamma$ had been studied in Refs. [16, 17].

Table 1. The branching fractions of $B_{s,d} \rightarrow \mu^+ \mu^-$ measured by the CMS and LHCb Collaborations [2] and predicted within the SM [1] with NNLO QCD [11] and NLO EW [12] corrections included.

	EX [2]	SM [1]	Deviations
$Br(B_d \rightarrow \mu^+ \mu^-)$	$(3.9^{+1.6}_{-1.4}) \times 10^{-10}$	$(1.06 \pm 0.09) \times 10^{-10}$	2.2σ
$Br(B_s \rightarrow \mu^+ \mu^-)$	$(2.8^{+0.7}_{-0.6}) \times 10^{-9}$	$(3.66 \pm 0.23) \times 10^{-9}$	1.2σ
$\frac{Br(B_d \rightarrow \mu^+ \mu^-)}{Br(B_s \rightarrow \mu^+ \mu^-)}$	$0.14^{+0.08}_{-0.06}$	$0.0295^{+0.0028}_{-0.0025}$	2.3σ

2 The Decay of $B_s^* (B_d^*) \rightarrow \mu^+ \mu^-$

Within the SM, effective Lagrangian related with $b\bar{s} \rightarrow \mu^+ \mu^-$ is given in Ref. [18, 19]

$$\mathcal{L} = \mathcal{N} \left[C_7^{eff}(\mu_f) \mathcal{O}_7^\gamma + C_9(\mu_f) \mathcal{O}_9^V + C_{10}(\mu_f) \mathcal{O}_{10}^A \right], \quad (2.1)$$

where $\mathcal{N} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{e^2}{4\pi^2}$, and the operators $\mathcal{O}_{7,9,10}$ read as

$$\mathcal{O}_7^\gamma = -\frac{2im_b(p_\mu^\nu + p_{\bar{\mu}}^\nu)}{(p_\mu + p_{\bar{\mu}})^2} (\bar{s} \sigma_{\rho\nu} P_R b) (\bar{\mu} \gamma^\rho \mu), \quad (2.2)$$

$$\mathcal{O}_9^V = (\bar{s} \gamma_\rho P_L b) (\bar{\mu} \gamma^\rho \mu), \quad (2.3)$$

$$\mathcal{O}_{10}^A = (\bar{s} \gamma_\rho P_L b) (\bar{\mu} \gamma^\rho \gamma_5 \mu), \quad (2.4)$$

where $P_L = (1 - \gamma_5)/2$, $P_R = (1 + \gamma_5)/2$. And the Wilson coefficients are $C_{7,9,10}(\mu_f) = (-0.316, 4.403 - 0.47i, -4.493)$ at $\mu_f = m_b = 4.5$ GeV [15]. The superscript γ , V , and A denote the contributions from photon, vector current, and axial vector current, respectively.

The relations between the quark level operators and the meson are described as

$$\langle 0 | \bar{s} \gamma^\mu b | B_s^*(q, \epsilon) \rangle = m_{B_s^*} f_{B_s^*} \epsilon^\mu, \quad (2.5)$$

$$\langle 0 | \bar{s} \sigma^{\mu\nu} b | B_s^*(q, \epsilon) \rangle = -i f_{B_s^*}^T (q^\mu \epsilon^\nu - \epsilon^\mu q^\nu), \quad (2.6)$$

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 b | B_s(q) \rangle = -i f_{B_s} q^\mu, \quad (2.7)$$

where the three decay constants f_{B_s} , $f_{B_s^*}$ and $f_{B_s^*}^T$ depend on the renormalization scale. Their relations have been studied in Ref. [20] in the heavy-quark limit. Ignoring the mass difference between B_s and B_s^* and the high order QCD corrections, we have

$$f_{B_s^*} = f_{B_s^*}^T = f_{B_s}. \quad (2.8)$$

Then the amplitudes of $B_s^* (B_s) \rightarrow \mu^+ \mu^-$ are [13]

$$\begin{aligned} \mathcal{M}(B_s^* \rightarrow \mu^+ \mu^-) &= f_{B_s^*} \frac{\mathcal{N}}{2} m_{B_s^*} \bar{\mu} / \epsilon \left[C_V^{eff} + C_{10} \gamma_5 \right] \mu, \\ \mathcal{M}(B_s \rightarrow \mu^+ \mu^-) &= -i f_{B_s} \mathcal{N} C_{10} m_{\mu} \bar{\mu} \gamma_5 \mu, \end{aligned} \quad (2.9)$$

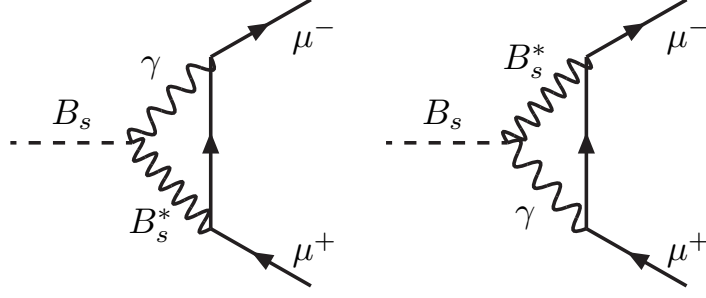


Figure 1. Feynman diagrams of $B_{s,d} \rightarrow B_{s,d}^* \gamma^* \rightarrow \mu^+ \mu^-$.

where

$$C_V^{eff} = C_9 + 2 \frac{m_b}{m_{B_s^*}} C_7^{eff}. \quad (2.10)$$

The helicity suppression factor m_μ^2/m_M^2 in the decay width is removed in the vector meson decay. Then we can get the decay widths of $B_s^*(B_s) \rightarrow \mu^+ \mu^-$

$$\begin{aligned} \Gamma(B_s^* \rightarrow \mu^+ \mu^-) &= \frac{G_f^2 \alpha_{em}^2}{96\pi^3} |V_{tb} V_{ts}^*|^2 \left(|C_{10}|^2 + |C_V^{eff}|^2 \right) m_{B_s^*}^3 f_{B_s^*}^2 \\ &\quad \left(1 + \mathcal{O}(m_\mu^2/m_{B_s}^2) \right) \\ \Gamma(B_s \rightarrow \mu^+ \mu^-) &= \frac{G_f^2 \alpha_{em}^2}{16\pi^3} |V_{tb} V_{ts}^*|^2 |C_{10}|^2 m_\mu^2 m_{B_s} f_{B_s}^2 \\ &\quad \left(1 + \mathcal{O}(m_\mu^2/m_{B_s}^2) \right) \\ \frac{\Gamma(B_s^* \rightarrow \mu^+ \mu^-)}{\Gamma(B_s \rightarrow \mu^+ \mu^-)} &= \frac{\left(|C_{10}|^2 + |C_V^{eff}|^2 \right) m_{B_s^*}^3 f_{B_s^*}^2}{6 |C_{10}|^2 m_\mu^2 m_{B_s} f_{B_s}^2} \left(1 + \mathcal{O}(m_\mu^2/m_{B_s}^2) \right) \end{aligned} \quad (2.11)$$

The ratio of decay width is about 700 for $B_s^{(*)}$ and $B_d^{(*)}$ both.

3 The impact of $B_s^*(B_d^*) \rightarrow \mu^+ \mu^-$ on $B_s(B_d) \rightarrow B_s^*(B_d^*) \gamma \rightarrow \mu^+ \mu^-$

Meanwhile, $B_{s,d}^*$ will impact on the leptonic decay of $B_{s,d}$ through the loop contribution $B_{s,d} \rightarrow B_{s,d}^* \gamma^* \rightarrow \mu^+ \mu^-$. The Feynman diagrams are given in Fig. 1. The vertex of $B_{s,d} \rightarrow B_{s,d}^* \gamma$ is given as the operator [21, 22]

$$\begin{aligned} \mathcal{M}_{B_s B_s^* \gamma} &= \sum_{q=s,b} \langle B_s^* \gamma | i e e_q \bar{q}(p_{\bar{q}}) \gamma_\mu q(p_q) | B_s \rangle \\ &= \sum_{q=s,b} \epsilon_\gamma^\mu p_\gamma^\nu \langle B_s^* | i e e_q \bar{q}(p_{\bar{q}}) \frac{i \sigma_{\mu\nu}}{2m_q} q(p_q) | B_s \rangle. \end{aligned}$$

We can simplify the matrix element $\langle B_s^* | \bar{q}(p) \sigma_{\mu\nu} q(p) | B_s \rangle$ with the procedure in Refs. [23, 24]¹,

$$\begin{aligned} \mathcal{M}_{B_s B_s^* \gamma} &= \sum_{q=s,b} \frac{-e e_q}{2m_q} \epsilon_\gamma^\mu p_\gamma^\nu \langle B_s^* | \bar{q}(p) \sigma_{\mu\nu} q(p) | B_s \rangle \\ &= i \epsilon_{\mu\nu\alpha\beta} \epsilon_\gamma^\mu p_\gamma^\nu \epsilon_{B_s^*}^\alpha p_{B_s^*}^\beta \sum_{q=s,b} \left(\frac{e e_q}{m_q} \right) \mathcal{J}. \end{aligned} \quad (3.1)$$

\mathcal{J} is related with the wave functions of B_s^* and B_s [23], and it is $\mathcal{J} = \langle B_s | j_0(p_\gamma r) | B_s^* \rangle \sim 1$ in the non-relativistic limit [25]. We can rewrite Eq. (3.1) with

$$\mathcal{M}_{B_s B_s^* \gamma} = i \frac{g_{B_s B_s^* \gamma}}{m_{B_s^*}} \epsilon_{\mu\nu\alpha\beta} \epsilon_\gamma^\mu p_\gamma^\nu \epsilon_{B_s^*}^\alpha p_{B_s^*}^\beta \quad (3.2)$$

Where the dimensionless vector-scalar-photon coupling constant $g_{B_s B_s^* \gamma}$ is related with the magnetic moments of b and s quarks. And the phase factor i is consistent with the amplitude of $\gamma^* \rightarrow VP$ in Ref. [26].

There are ultraviolet (UV) logarithmical divergences in the evaluation of loop integrals. Then we introduce a cut off regularization scheme for the UV divergence integral

$$\begin{aligned} &\int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q_i^2 - m_i^2)(q_j^2 - m_j^2)} \\ &\rightarrow \int \frac{d^4 q}{(2\pi)^4} \left[\frac{1}{(q_i^2 - m_i^2)(q_j^2 - m_j^2)} \right. \\ &\quad \left. - \frac{1}{(q_i^2 - (m_i + \Lambda)^2)(q_j^2 - (m_j + \Lambda)^2)} \right], \end{aligned} \quad (3.3)$$

where $i, j = B_s^*, \gamma$ or μ , and q_i corresponds the momentum of i in loop. $\Lambda \ll M_W$ for the amplitude is UV finite when W boson are involved. Since the hadronic contribution will be suppressed when $\sqrt{q_j^2} - m_j \gg \Lambda_{QCD}$, Λ is about several Λ_{QCD} . The cut off regularization scheme is similar Pauli-Villars regularization scheme but acts on two propagators. The Pauli-Villars regularization scheme of the UV divergence integral is the same with the form factor \mathcal{F} which is introduced in the $B_s B_s^* \gamma$ vertex in Ref. [27]

$$\mathcal{F} = \left(\frac{\Lambda^2 - m_{B_s^*}^2}{\Lambda^2 - q_{B_s^*}^2} \right), \quad (3.4)$$

for

$$\frac{1}{q_{B_s^*}^2 - m_{B_s^*}^2} \mathcal{F} = \frac{1}{q_{B_s^*}^2 - m_{B_s^*}^2} - \frac{1}{q_{B_s^*}^2 - \Lambda^2}. \quad (3.5)$$

But here it acts on the UV divergence term only and the two propagators. Then the soft contribution will be maintained in our calculation.

¹The decay constants defined of vector meson in Eq. (2.5) is different with Eq.(44) in Ref. [23] with an additional i .

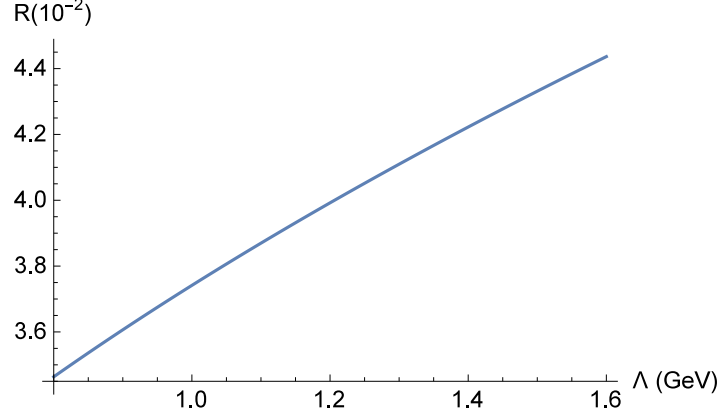


Figure 2. $R(\Lambda)$ of $B_s \rightarrow \mu^+ \mu^-$ defined in Eq.(3.6) as a function of the cut off energy.

The amplitude from $B_s \rightarrow B_s^* \gamma^* \rightarrow \mu^+ \mu^-$ can be written as

$$\mathcal{M}(B_s \rightarrow B_s^* \gamma^* \rightarrow \mu^+ \mu^-) = i e \mathcal{N} g_{B_s B_s^* \gamma} R(\Lambda) C_V^{eff} f_{B_s^*} m_\mu \bar{\mu} \gamma_5 \mu. \quad (3.6)$$

m_μ reappears in the amplitude of leptonic decay of scalar meson. The factor $R(\Lambda)$ serves as a function of the high energy cut is shown in Fig.2. More information about the factor $R(\Lambda)$ are given in the Appendix.

Only the C_{10} term is taken into account in the calculation of the NNLO QCD [11] and NLO EW [12] corrections of $B_{s,d} \rightarrow \mu^+ \mu^-$ within the SM [1]. But the contribution from B_s^* is missed in the previous calculation, which is also related with C_7 and C_9 terms.

Compared with Eq.(2.9), the previous amplitude is added a factor F ,

$$\begin{aligned} F(B_s^*) &= \frac{\mathcal{M}(B_s \rightarrow B_s^* \gamma^* \rightarrow \mu^+ \mu^-)}{\mathcal{M}(B_s \rightarrow \mu^+ \mu^-)} \\ &= -\frac{C_V^{eff} f_{B_s^*}}{C_{10} f_{B_s}} e g_{B_s B_s^* \gamma} R(\Lambda) \end{aligned} \quad (3.7)$$

We can estimate $g_{B_s B_s^* \gamma}$ in several ways, including the heavy-quark and chiral effective theories [28, 29] with the radiative and pion transition widths of D^{*+} , light cone QCD sum rules [30, 31], and the radiative M1 decay widths of $B_s^* \rightarrow B_s \gamma$ from potential model [25, 32]. The radiative M1 decay width of $B_s^* \rightarrow B_s \gamma$ is

$$g_{B_s B_s^* \gamma} = -m_{B_s^*} \left(\frac{12\pi}{E_\gamma^3} \Gamma(B_s^* \rightarrow B_s \gamma) \right)^{1/2}. \quad (3.8)$$

The predicted M1 widths are 0.15–400 eV and 10–300 eV for $B_s^* \rightarrow B_s \gamma$ and $B_d^* \rightarrow B_d \gamma$, respectively [22, 24, 25, 32, 33].

4 Numerical Result

The parameters in the numerical calculation are chosen as [34]

$$\begin{aligned}\Lambda &= 1.2 \text{ GeV}, \\ m_b &= 4.2 \text{ GeV}, \\ \alpha_{em} &= 1/137.\end{aligned}\tag{4.1}$$

The branch fraction of $B_{s,d}^*$ weak decay is much less than the M1 decay, $\Gamma_{tot}(B_{s,d}^*) \approx \Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma)$. We can get the ratio

$$\begin{aligned}\frac{Br(B_s^* \rightarrow \mu^+ \mu^-)}{Br(B_s \rightarrow \mu^+ \mu^-)} &= (0.34 \pm 0.03) \times \frac{\text{eV}}{\Gamma(B_s^* \rightarrow B_s \gamma)}, \\ \frac{Br(B_d^* \rightarrow \mu^+ \mu^-)}{Br(B_d \rightarrow \mu^+ \mu^-)} &= (0.33 \pm 0.03) \times \frac{\text{eV}}{\Gamma(B_d^* \rightarrow B_d \gamma)}.\end{aligned}\tag{4.2}$$

The main uncertainty resulting from the value of $f_{B_{s,d}}^*$. The distribution of the dimuon invariant mass measured by CMS and LHCb Collaborations in Ref. [2] should include the contributions of $B_{s,d}^* \rightarrow \mu^+ \mu^-$. If $\Gamma(B_s^* \rightarrow B_s \gamma) = 1 \text{ eV}$ [25], we can get $\frac{Br(B_s^* \rightarrow \ell^+ \ell^-)}{Br(B_s \rightarrow \mu^+ \mu^-)} = 0.34$ for $\ell = e, \mu$. Then $B_s^* \rightarrow e^+ e^-$ may be searched by CMS and LHCb experiments with larger data samples.

In another hand, if $\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma) \sim 100 \text{ eV}$, we can find that the amplitude of $B_{s,d} \rightarrow \mu^+ \mu^-$ will be modified by the contributions of $B_{s,d}^*$ with a factor

$$F(B_{s,d}^*) = (0.011 \pm 0.006) \sqrt{\frac{\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma)}{100\text{eV}}},\tag{4.3}$$

The main uncertainty resulting from the value of Λ . Then the new predictions of $\Gamma(B_{s,d} \rightarrow \mu^+ \mu^-)$ are

$$\begin{aligned}Br(B_s \rightarrow \mu^+ \mu^-) &= (36.6 \pm 2.3) \times \left(1 + (0.023 \pm 0.012) \sqrt{\frac{\Gamma_{tot}(B_s^*)}{100\text{eV}}} \right) \times 10^{-10}, \\ Br(B_d \rightarrow \mu^+ \mu^-) &= (10.6 \pm 0.9) \times \left(1 + (0.023 \pm 0.012) \sqrt{\frac{\Gamma_{tot}(B_d^*)}{100\text{eV}}} \right) \times 10^{-11}.\end{aligned}$$

If $\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma) = 200 \text{ eV}$, this factor will increase the decay width $\Gamma(B_{s,d} \rightarrow \mu^+ \mu^-)$ by a factor $(3.3 \pm 1.7)\%$, which is about a factor of 10 larger than the neglect NLO EW correction factor 0.3% at the decay width in Ref. [1]. And the corresponded $g_{B_{s,d}B_{s,d}^*\gamma} = -1.5$, about a factor of 15 larger than the $e_q e = -1/3\sqrt{4\pi\alpha_{em}} = -0.10$. The $\Gamma(B_{s,d}^* \rightarrow B_{s,d}\gamma)$ may be measured through two-body decay $B_{s,d}^* \rightarrow J/\psi + M$ by CMS and LHCb.

5 Summary

In summary, $B_{s,d}^* \rightarrow \mu^+ \mu^-$ in the dimuon distributions and the hadronic contribution $B_{s,d} \rightarrow B_{s,d}^* \gamma \rightarrow \mu^+ \mu^-$ are studied here. The $\mu^+ \mu^-$ decay widths of vector mesons $B_{s,d}^*$ are about a factor of 700

larger than the corresponding scalar mesons $B_{s,d}$. The obtained ratio of the branching fractions $Br(B_{s,d}^* \rightarrow \mu^+ \mu^-) / Br(B_{s,d} \rightarrow \mu^+ \mu^-)$ is about $0.3 \times \text{eV} / \Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma)$. At the same time, the hadronic contribution $B_{s,d} \rightarrow B_{s,d}^* \gamma \rightarrow \mu^+ \mu^-$ is calculated too. The amplitude of $B_{s,d} \rightarrow \mu^+ \mu^-$ is enhanced by a factor of $0.01 \sqrt{\Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma) / 100 \text{ eV}}$. If we choose $\Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma) = 2 \text{ eV}$, the branching fractions of the vector mesons to lepton pair are 5.3×10^{-10} and 1.6×10^{-11} for B_s^* and B_d^* respectively. If we choose $\Gamma(B_{s,d}^* \rightarrow B_{s,d} \gamma) = 200 \text{ eV}$, the updated branching fractions of the scalar mesons to muon pair are $(3.78 \pm 0.25) \times 10^{-9}$ and $(1.09 \pm 0.09) \times 10^{-10}$ for B_s and B_d respectively. Further studies on $B_{s,d}^*$ are usefully here, including dielectron decay, two-body decay with J/ψ , and so on.

Acknowledgments

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6 Appendix: $R(\Lambda)$

$R(\Lambda)$ of $B_s \rightarrow B_s^* \gamma^* \rightarrow \mu^+ \mu^-$ defined in Eq.(3.6) is given as:

$$\begin{aligned}
 R(\Lambda) = & \frac{1}{32\pi^2 m_{B_s}^2 m_\mu^2} \left\{ 3m_{B_s}^2 m_\mu^2 - 2m_\mu^2 (m_{B_s}^2 - m_{B_s^*}^2)^2 C_0(m_{B_s}^2, m_\mu^2, m_\mu^2, m_{B_s^*}^2, 0, m_\mu^2) \right. \\
 & + m_{B_s}^2 (m_\mu^2 - m_{B_s^*}^2) \left(B_0(0, m_\mu^2, m_{B_s^*}^2) - B_0(0, (\Lambda + m_\mu)^2, (\Lambda + m_{B_s^*})^2) \right) \\
 & + \left(m_{B_s}^2 (2m_\mu^2 + m_{B_s^*}^2) + 2m_\mu^2 m_{B_s^*}^2 \right) \left(B_0(m_\mu^2, m_\mu^2, m_{B_s^*}^2) - B_0(m_\mu^2, (\Lambda + m_\mu)^2, (\Lambda + m_{B_s^*})^2) \right) \\
 & \left. + m_\mu^2 (3m_{B_s}^2 - 2m_{B_s^*}^2) \left(B_0(m_\mu^2, 0, m_\mu^2) - B_0(m_\mu^2, \Lambda^2, (\Lambda + m_\mu)^2) \right) \right\} \quad (6.1)
 \end{aligned}$$

The scalar functions B_0 and C_0 are given in Ref. [35–37]. As a numerical fit between $0.5 - 2 \text{ GeV}$, we can get

$$R(\Lambda) = 0.022 + 0.062 \times \ln\left(\frac{\Lambda + m_{B_s}}{m_{B_s}}\right). \quad (6.2)$$

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